From: **nicholas wheeler** nwheeler@reed.edu
Subject: Superoscillations Again, + Michael Berry + REC
Date: December 26, 2017 at 4:09 PM
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Hello again,

Doubtless some of you have already trashed the material I distributed on 22 December, but in others of you it may have sparked an interest that, because of holiday distractions and the press of other work, you have as yet had no opportunity to acknowledge. It is to you latter that I address the following.

On 24 December I sent to Michael Berry, which whom I am (thanks to Richard Crandall) remotely acquainted, this variant of the material already distributed to you (a couple typos now corrected, + the first of the Addenda + two additional figures)

Superoscillation Theory 4.pdf

Superoscillation Figures

Within hours I received this response:

Dear Nicholas,

Thanks for your message. I agree that once you get the idea of superoscillations, as you clearly have, it's fun to play with pictures.

As you say, the subject has advanced a great deal since the 'Beethoven at 1Hz' days, and many people are now writing about this mathematical phenomenon and its physical applications. In case you are interested, I attach a list of my own superoscillation-related papers.

Your title could have read the opposite: 'When a part [i.e. an interval of the *x* axis] vibrates faster than the [band-limited]

whole'. I find this an amusing circularity.

I learned about superoscillations when Aharonov visited Bristol in 1990, not at the 1992 conference (which was in South Carolina not Bristol).

Only (too) much later did I realise that there was a sense in which already in the 1970s I was familiar with what I later called superoscillations, in the form of phase singularities in waves (a.k.a. wavefront dislocations, wave vortices, nodal points and lines...). 'Optical vorticulture' and superoscillations are now fully integrated, and a number of my papers now reflect this (e.g. #404, #412, #457, #463).

The most dramatic mathematical application of superoscillations that I know is that they can reproduce fractals - see my paper #499.

Mindful of the renewed interest in the subject, I now have a five-lecture presentation of the subject. I've given it in a number of places, starting in 1995 in Chapman where I'm a a member of their Institute for Quantum Studies (though I don't go there as often as Aharonov, and I haven't yet met Ahmed Sebbar). In mid-January, I'll give it in Tel Aviv.

I also miss Richard Crandall, though of course my acquaintance with him was more superficial than yours.

Best wishes, Michael

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I have incorporated some of that information into my second Addendum. In 2012 Berry presented me with a thumb drive containing the first 449 of his papers. His first two references were to "Optical lattices with PT symmetry are not transparent," J. Phys. A **41**, 244007 (2008) and "Natural superoscillations in monochromatic waves in D dimensions," J. Phys. A **42**, 022003 (2009). For the particulars of ##457, 463 & 499 I will have to ask him. Berry writes prolifically, but that large number includes many book reviews and casual little essays, yet even some of the latter (treating subjects like the Aharonov-Bohm effect) are of substantial significance. And, of course, more than his fair share of papers are of enduring fundamental significance.

Whenever my work intersects that of Berry (and in many other connections also!) I think of Richard Crandall. Whose death almost exactly five years ago led me at that time to construct a Mathematica notebook, of which this is a pdf version:

Richard's Last Problem.pdf

It was I who notified Stephen Wolfram of Richard's death, and Wolfram who soon thereafter shared with me the draft his REC obituary (reprinted in Wolfram's *Idea Makers: Personal Perspectives on the Lives & Ideas of Some Notable People* (2016)). When I sent to Wolfram the preceding notebook he responded that "It pleases me to see Mathematica used in much a way." It was again I who brought the sad news to the attention of Berry. When, in the aftermath of REC's demise, I sent the above notebook to Berry he responded (4 February 2013)

Dear Nicholas,

Thank you for your kindness in sending your Mathematica notebook tribute to Richard, with its ingenious and (to me at least) unfamiliar matrix manipulations.

I've dipped my toe into somewhat similar waters in the context of polarization optics. You might find my paper 361 amusing (it's on the thumb drive you have, also my home page), also possibly 269, touching on different representation questions.

Later this week I'll be in Boston, and might meet Stephen Wolfram; you've probably seen his tribute to Richard - rather moving, in a way I don't associate with Stephen.

In the notebook you again mention the dinner at Reed so many years ago. I can only apologise for not remembering, weakly pleading advancing senility and a travel schedule (then and now) that leaves me more delocalised than an EPR-Bell quantum state.

Best wishes, Michael

On 15 January he had written

Dear Nicholas,

I'm not sure if I sent you the little appreciation of Richard Crandall (below), that I have submitted to Rudolph van der Merwe for the memorial page they are creating.

Also, you might care to look at http://iopscience.iop.org/1367-2630/15/1/013026, where if you click on Full text PDF you will see that we dedicated the paper to Richard's memory.

Best wishes, Michael

For Richard Crandall memorial publication

I knew of Richard as a scientist for many years, starting with his book 'Mathematica for the Sciences'. But only in 2012, following stimulating encounters with him in Portland and Vancouver and a brief correspondence, did I begin to appreciate the vast range and flair of his contributions: image processing, rendering mathematics as sound, high-accuracy arithmetic as a tool for mathematical exploration, factorization, lattice sums, quantum physics... He was a most original scientific man, intensely driven by the need to understand. How tragic that he died so prematurely, still in his creative prime. And how I regret the missed opportunity to enjoy more of his scientific companionship, with our newly discovered intellectual resonance, and his poignant last sentence to me: "We must write a paper together".

Michael Berry, Physicist, University of Bristol, United Kingdom

In early February Berry had occasion to mention to me a couple of papers (``Black polarization sandwiches are square roots of zero,'' J. Opt. A: Pure Appl. Opt. **6**, S24-S25 (2004) and "Nonpropagating string excitations," AJP **62,** 121-123 (1998)) that led me to send to him on 5 February a copy of my 150-page "Ellipsometry" (1998), which treats a number of related ideas that occur in optics and classical/quantum mechanics. The very next day (typically, but by which time he could not actually have read my long and excessively detailed paper) he responded

Dear Nicholas,

Thank you. The way you present all this standard material [to which I took some exception, since some of my material was—at least to me—original] is pedagogy at its most inspired. I have thought similarly about David Griffiths's writings (which I have read because he does publish!), so Reed must be a truly wonderful place to learn. I hope your students appreciate this.

One point about Poincaré. We appreciate his sphere as fundamental, and the equivalent complex variable description (South-pole stereographic projection of points on the sphere, or, equivalently, a+/a-, namely the ratio of spinor coefficients in a circular-polarization basis) as an occasionally useful alternative. But in his comprehensive Lectures on Optics, he uses the complex representation almost throughout, introducing his sphere only near the end, almost as an afterthought and in the context of a not very interesting little application.

Best wishes, Michael

Best wishes, Michael

I include reference to that exchange because of its encouraging reference to what we at Reed have tried to do.

Best to you all. I promise not to write at such length again!

Nick